Transport, Noise, and Conservation in the Electron Gas: How to Build a Credible Mesoscopic Theory

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Electron transport in metallic systems is governed by four key principles of Fermi-liquid physics: (i) degeneracy, (ii) charge conservation, (iii) screening of the Coulomb potential, and (iv) scattering. They determine the character of metallic conduction and noise at mesoscopic scales, both near equilibrium and far from it. Their interplay is described by *kinetic theory*, the serious method of choice for characterizing such phenomena. We review microscopic kinetics for mesoscopic noise, and in particular its natural incorporation of the physics of Fermi liquids. Kinetic theory provides a strictly conservative, highly detailed description of current fluctuations in quantum point contacts. It leads to some surprising noise predictions. These show the power of a model that respects the microscopic conservation laws. Models that fail in this respect are incorrect.

1. INTRODUCTION

1.1 History

Sustained, vigorous progress marks 100 years of thought on a cornerstone of modern electronics: the physics of metallic charge transport. Its flowering began with the classical insights of Boltzmann in the 19th century and of Drude and Einstein early in the last one. In the 1930s Sommerfeld and Bloch instituted the quantum description of bulk metallic conduction [1]. That development culminated in the microscopically robust *Fermi-liquid picture* [2–4], proposed by Landau and Silin late in the 1950s. [1]

Fresh horizons have now opened up through the vision of mesoscopic transport as quantum-coherent transmission. This important innovation is credited to Landauer's foresight [5], and has been deepened and extended since then by Beenakker, Büttiker, Imry, and many others [6–14]. Its achievements have been impressive.

As well as their novel emphasis on coherent scattering, the modern theories of conduction advocate a second major shift, one that is logically independent of the mechanism for transport (quantum-coherent or otherwise). This is claimed to solve the subtle problems of *open boundary conditions* [16–19], central to conduction in a real mesoscopic system [9]. It is intended to supplant the long-dominant picture of charge flow as *drift*.

In drift, the current is the collective average response to which every carrier contributes. It is the effect of an external cause, the applied voltage. Over against drift, the new mesoscopic transport revisits the notion of charge flow as purely a kind of *diffusion*. Here, it is the current that is regarded as externally supplied [9]. Introduction of current into the system sets up a virtual density imbalance between the carrier reservoirs interconnected by the transmissive device. The observed voltage drop is merely a by-product of that virtual imbalance.

Figure 1 illustrates the two viewpoints; diffusion and drift are seen in their seemingly contrasting physical roles. This simple shift of perspective, from drift to diffusion, has been extraordinarily successful in predicting mesoscopic transport phenomena. These have been carefully documented and explained [9–14].

The explanatory simplicity and consequent attractiveness of diffusive phenomenologies does not mean, however, that microscopically based analyses of transport and noise have become redundant. Microscopic methods, built upon the legacy that runs from Boltzmann to Landau, are pursued with unabating vigor [15–29]. That said, phenomenological simplicity has never been a fool-

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[1] It is important to keep in mind, for the rest of this paper, that standard Fermi-liquid theory was never conceived as a theory of the bulk. It has *always* addressed metallic transport at *all* scales, including the mesoscopic one. Indeed, its pedigree derives from extreme quantum many-body problems, such as nuclear matter and liquid ^{3}He [3].

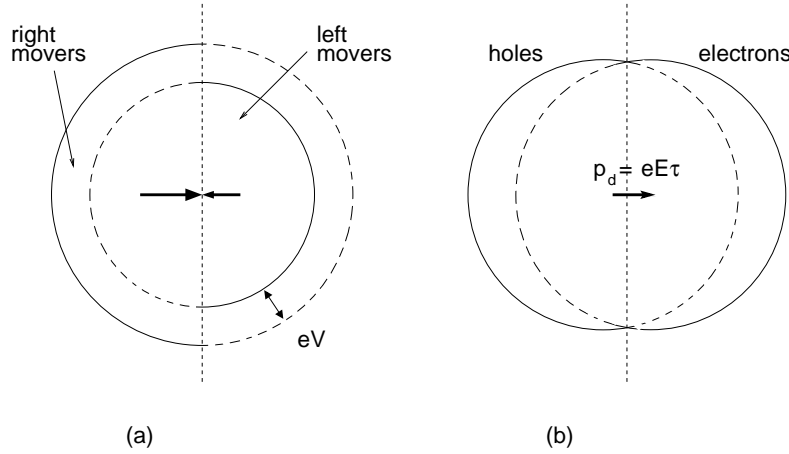proof guide to theoretical depth and correctness. The situation in mesoscopic physics is no different.

FIG. 1. Diffusive and drift concepts of mesoscopic transport, compared. (a) Diffusion. An applied electromotive force eV defines a mismatch between the quasi-Fermi energies of degenerate electrons at the source and drain. Only "right movers" at the higher-energy source lead contribute to current flow; only "left movers" at the lower-energy drain lead contribute to the current counterflow. The physical current is phenomenologically identified with their difference. This pseudodiffusive current "generates" eV if and only if one additionally assumes the validity of Einstein's relation between diffusion and conductance. *All carriers are at equilibrium; their role in transport is passive.* There is no electron-hole symmetry in pseudodiffusive transport [18]. (b) Drift. All of the carriers that fill states in the Fermi sea feel, and respond to, the external driving force eE. Each gains average momentum $p_d = eE\tau$ by accelerating ballistically during a mean time τ before rescattering. The flux of carriers in deeper-lying filled states is canceled by opposing filled states. Only those electron states *kinematically matched* to holes, within a shell of thickness $p_d v_F$ at the Fermi surface, contribute to the physical (drift) current. *The volume of the Fermi sea remains invariant regardless of eE.* The volume is rigidly fixed by the *equilibrium* Fermi energy. There is automatic electron-hole symmetry in drift transport [18].

For a mesoscopic theory's credibility, only two questions count:

- Does the theory fully respect *all* of the essential physics of the interacting electron gas [3]?

- If not, *why* not? (Some discussion of this is in References [25,30,31].)

Our goal is straightforward. We restate, and elaborate, a plain theoretical fact. If a noise model is truly microscopic – faithful to the long-established and completely orthodox procedures of kinetics and electron-gas theory [3,15,20] – then it must, and does, produce reliable predictions at mesoscopic scales. These may be quite surprising.

Microscopically based descriptions, for instance kinetic ones, outstrip the scope of low-field phenomenologies to access the strongly nonequilibrium regime. Equally important is the fact that only a reliable microscopic foundation can support the well controlled approximations that are always needed to turn a generic theory into a powerful, practical design tool for novel electronics.

The heart of any kinetic approach is conservation. Microscopic conservation implies that diffusion and drift manifest as complementary but *interlocking* effects in the physics. They are in no sense mutually exclusive. This crucial point needs a closer look.

1.2 Drift or Diffusion?

Before setting out the plan of our paper, we briefly address the folklore that transmissive-diffusive models are more "physical" than (and somehow superior to) wholly kinetic descriptions of mesoscopic transport. For *uniform* systems, there is a formal congruence between "pure" drift and "pure" diffusion. They connect via the Einstein relation [34,35] which links σ, the low-field conductivity of a metal, to D, its equilibrium diffusion constant:

$$\sigma \equiv e^2 D \frac{\partial n}{\partial \mu} \qquad (1)$$

at carrier density n and chemical potential μ. Substitution of diffusion for (weak-field) conductance is justified when the system's shortest scattering mean free path is much less than its length. However, it is claimed that this clearly semiclassical Ansatz can be extended even to quantum-coherent mesoscopics [12].

The conductivity quantifies the coarse-grained *single-particle* current response; σ is accessible through the current-voltage characteristic. The diffusion constant is a fine-grained *two-body* response, and its structure is intimately tied to current fluctuations; D too is observable, for example via time-of-flight methods that are essentially two-point correlation measurements [36]. Equation (1) clearly shows that diffusion and drift go hand-in-hand; it is not an either-or situation.

Einstein's relation between conductivity and diffusion brings to the fore a central theme, namely the *underlying unity of transport and fluctuations* (noise). This unity, which is fundamentally microscopic, is embodied in the fluctuation-dissipation theorem (the Einstein relation is a special case). It establishes the proportionality of dissipative transport to the fluctuations inherent in the structure. Such a theorem can never be proved heuristically [30].

This is the crucial point. Diffusive phenomenologies are forced to invoke the fluctuation-dissipation theorem as an external assumption. It is their *only means* to justify, in an intuitive way, the linear current-voltage characteristic on which they absolutely rely. A transport model that chooses to favor diffusion, merely for intuitive reasons, denies the core *microscopic unity* of noise and conductance. The fluctuation-dissipation relation is then no longer a prescriptive, first-principles constraint on the possible physics of the problem. Instead it is reduced to a highly compliant, imaginative guiding "rule"; one that can be molded to any set of favorite preconceptions.

For noise, diffusive (or, more accurately, pseudodiffusive) descriptions invariably take this linear theorem on faith. This is so that the current-current correlator can be adjusted, by hand, to force it to fit the conductance. Such maneuvers are necessary only because, quite unlike microscopic theories (the Kubo formalism [37] is a good example), diffusive phenomenologies cannot express – and thus compute – their correlators from first principles.

The transmissive-diffusive models lack a formal basis for deriving the fluctuation-dissipation theorem [30,31]. That result is provable only within a microscopic description, embedded in *statistical mechanics* [37], or else in *kinetic theory* [20]. Models of the Landauer-Büttiker-Imry class share little, if any, of that essential machinery.

Few mesoscopic systems are truly homogeneous on the length scale over which transport unfolds. Generally, the mode of electron transfer through a nonuniform channel is not by real-space diffusion alone, or by drift alone (that is: diffusion in velocity space). Actual mesoscopic transport is some combination of drift and diffusion, physically conditioned by the nonuniformities specific to the system. For instance, the electron gas in a III-V heterojunction quantum well [38] is extremely nonuniform in the direction of crystal growth, normal to the plane of conduction. (This also leads to strong quantum confinement and to marked suppression of the fluctuations for the two-dimensional carriers [26].) To insist that one transport mode is absolutely dominant is to risk distorting the real physics.

Only a description that treats diffusion and drift on an equal footing, favoring neither one process nor the other *ad hoc*, is able to span in a unified fashion the complete range of transport and noise physics. Orthodox kinetic theory [20], coupled with precise microscopic knowledge of the electron gas [3], provides exactly that description. It accommodates both *nonuniform-field effects* and *nonequilibrium response.*

Finally we recall that weak-field approaches of the transmissive-diffusive kind tend to assume that the metallic electron gas is well described as a group of free, noninteracting fermions subject only to elastic scattering [9]. It means that self-consistent collective screening – ever preeminent in the electron gas – is regarded as a secondary perturbation (if, in fact, it is believed to matter at all). Such theories are not set up to describe strongly nonuniform Coulomb correlations [26], any more than they can treat the strongly nonequilibrium domain where dissipative *inelastic* collisions rule explicitly [28].

1.3 Issues for Review

To venture into the important regimes of high-field transport and Coulomb correlations, much more is demanded of a mesoscopic theory than is deliverable by current descriptions [9–14]. Among the sea of literature, it is still unusual to find theories of metallic conduction that explicitly adopt clear and firmly validated microscopic methods. At and beyond the low-field limit, a small but growing number of kinetic approaches exists [21–23,25–29], designed to answer the often-stated need [14] for new mesoscopic approaches, especially away from equilibrium.

For novel technologies, if not for the sake of fundamental physics alone, closure of this knowledge gap is a significant task. Our own endeavors are detailed in Refs. [25–28,30–33]. The present work is an up-to-date survey of that research.

In Section 2 we briefly introduce the two primary results of our exactly conserving kinetics: (i) *thermal scaling* and (ii) *Coulomb-induced suppression* of nonequilibrium fluctuations in a mesoscopic metallic conductor. We discuss their physical meaning, and their place in a coherent understanding of mesoscopic noise. While these core concepts are easy to state, their formal basis requires elaboration. This is given in Sec. 3; we cover the roles of *degeneracy*, *conservation*, and *screening*. For that we draw on the Landau-Silin equation of motion [3], itself

an extension of Boltzmann transport to charged Fermi liquids. In Sec. 4 we turn to a significant application: nonequilibrium ballistic fluctuations in one dimension. Our strictly conservative kinetic model leads to some surprises. We state our conclusions in Sec. 5.

2. PHYSICS OF THE BOUNDARY CONDITIONS

2.1 Ground Rules for Nonequilibrium Transport

In this Section we discuss two elementary, and indispensable, boundary constraints on an externally driven open conductor. They are [25]:

- *Global charge neutrality* over the conductor, its interfaces, and the connected source and drain reservoirs. Gauss' theorem implies *unconditional* global charge neutrality; that is, a neutrality that is absolutely independent of the dynamics within the active body of the device.

- *Local thermodynamic equilibrium* in each source and drain lead interfacing with the device. Energetic stability means that each of these local reservoir equilibria is also *unconditional* and independent of internal dynamics.

Both of these are universally understood as crucial for transport in open systems, yet their microscopic consequences seem not to be understood as well. We expand on them.

Figure 2 shows a generic two-terminal situation. The device is in intimate electrical contact with its two stabilizing reservoirs while a closed loop, incorporating an ideal generator, sustains a controlled current between drain and source. The system attempts to relax via net charge displacement across the source and drain. The induced potential – Landauer's resistivity dipole [5] – is the response. Equivalently, a closed loop with an ideal battery in series with the structure can be created, exerting a controlled electromotive force (EMF) *locally* across the active region [19]. The response is the carrier flux induced in the loop.
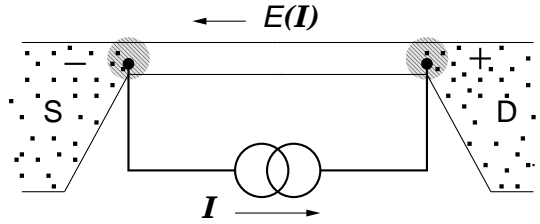

FIG. 2. An idealized mesoscopic conductor. Its diffusive leads (S, D) are in unconditional equilibrium. A paired source and sink of current I at the boundaries explicitly drives the transport. Local charge clouds (shaded) are induced by the active influx and efflux of I. These are regions of vigorous and dynamic competition among the current-driven excitation of carriers, their elastic *and inelastic* dissipative relaxation, and strong Coulomb screening from the stabilizing lead reservoirs. Together, these competing effects establish the self-consistent dipole potential $E(I)L$ across distance L between drain and source; that potential is the electromotive force in the driven system.

2.2 Charge Conservation for Open Systems

In either of the two scenarios (fixed current or fixed EMF), the specific action of the external flux sources and sinks ensures that electronic transport through the open system conserves charge [17]. Entry and exit of the current in a mesoscopic conductor cannot be treated by vague appeals to asymptotic equilibrium [9]. That is because entry and exit of the current is always a dynamic nonequilibrium process.

To guarantee global gauge invariance, all sources and sinks *must be considered explicitly* as part of the dynamical description of the transport [17]. If not, the price is clear. It is the loss of charge conservation, and an ill-conceived model.

Under all circumstances, the current sources and sinks, and the EMF, are *localized* inside a finite volume that also encloses the conductor [18,19]. This, like global neutrality, is a necessary consequence of gauge invariance [17]. Outside the active volume, the undisturbed electron population within each lead (stabilized by its compensating positive background) always remains charge-neutral and pins the local Fermi level within that lead. It means that the nonequilibrium carriers in the active, and finite, conducting channel have to reconnect smoothly to the *invariant* local equilibrium state beyond the interfaces [26].

The reservoir equilibria (each one locally proper to its lead) remain totally unaffected by the transport dy-

namics. None of the local density-dependent quantities within the leads, including their fluctuations, ever changes. None ever responds to the possibly extreme conditions in the driven device. This proves to be a formidable constraint on what can happen inside.

2.3 Constraint on the Total Carrier Number

Let $f_{\mathbf{k}}(\mathbf{r}, t)$ be the time-dependent electron distribution for wave vector $\mathbf{k}$, at point $\mathbf{r}$ in the active region. Spin and subband labels are understood (for simplicity we take only twofold spin degeneracy). From the microscopic object $f_{\mathbf{k}}(\mathbf{r}, t)$, all the physical one-body properties can be calculated, such as the mean electron density $n(\mathbf{r}, t)$ and the current density $\mathbf{J}(\mathbf{r}, t)$.

If N is the total number of carriers within the region, of volume Ω say, [2] then a sum of local momentum states over the entire active region, of dimension $\nu = 1, 2$, or 3, leads to

$$\int_{\Omega} d\mathbf{r} \int \frac{2d\mathbf{k}}{(2\pi)^{\nu}} f_{\mathbf{k}}(\mathbf{r}, t) = N = \int_{\Omega} d\mathbf{r} \int \frac{2d\mathbf{k}}{(2\pi)^{\nu}} f_{\mathbf{k}}^{\mathrm{eq}}(\mathbf{r}) \quad (2)$$

where $f_{\mathbf{k}}^{\mathrm{eq}}$ is the equilibrium distribution. The mean total carrier number is constant and remains fully compensated by the nonparticipating positive background, integrated over Ω.

Equation (2) makes a straightforward statement. Gauss' theorem implies – unconditionally – that the device remains overall neutral at any driving field. This is true if and only if the inner active region is efficiently screened from the macroscopic leads by the electron gas at the interfaces [29]. Mean-field screening (Poisson's equation) thus ensures the leads' (local) neutrality at all times, while the asymptotic equilibrium of each lead ensures that the total volume Ω, where nonequilibrium processes take place, is fixed and finite.

Whether in equilibrium or not, we have the principle that

- *Within the active mesoscopic structure, the mean total number of mobile carriers is invariant.*

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[2]It is absolutely essential to include the interface regions (the buffer zones where all the fringing fields are extinguished by screening) as part of the active volume of the driven device.

[3]Examples are *quasicontinuous* energy exchange with phonons in the thermal bath of the lattice (generating thermal noise), and *discrete* Poissonian injection/extraction of carriers by the external sources/sinks of current (generating shot noise).

[4]The microscopic structure of Δf^{eq} is richer than its simple statistical mechanics definition suggests. It is better to recall its kinetic origin as a quantum-correlated *electron-hole excitation* taken in its long-wavelength static limit [3]:

Belying its almost self-evident nature, this rule has profound implications for the fluctuations of the nonequilibrium state.

2.4 Constraint on Total Fluctuation Strength

Random external perturbations give rise to a persistent fluctuation background. This displaces the instantaneous distribution $f_{\mathbf{k}}(\mathbf{r}, t)$ from its steady-state ensemble average. The *same* external stochastic processes [3] act on the channel both at equilibrium and when it is driven by an injected current (or by a battery-generated EMF).

Let $\Delta N \equiv k_{\mathrm{B}} T \partial N / \partial \mu$ be the mean-square thermal number fluctuation. Then Gauss' theorem acts as a constraint on Eq. (2) for N, taking note that the latter is (potentially) a dynamical quantity. As a result, global neutrality enforces a *fluctuation* counterpart to the sum rule of Eq. (2). This involves, in the one relation, the mean-square thermal fluctuation $\Delta f(t)$ of the single-particle distribution $f(t)$, and its basic equilibrium form Δf^{eq}:

$$\sum_{\alpha} \Delta f_{\alpha}(t) = \Delta N = \sum_{\alpha} \Delta f_{\alpha}^{\mathrm{eq}}. \quad (3)$$

For brevity, we have condensed the notation. We now use the composite state-labels $\alpha \equiv (\mathbf{k}, \mathbf{r})$, $\alpha' \equiv (\mathbf{k}', \mathbf{r}')$ and so on, while the generalized sum (with spin degeneracy) is defined by

$$\sum_{\alpha} \cdots \equiv \sum_{\mathbf{r}} \Omega(\mathbf{r}) \sum_{\mathbf{k}} \frac{2}{\Omega(\mathbf{r})} \cdots \equiv \int_{\Omega} d\mathbf{r} \int \frac{2d\mathbf{k}}{(2\pi)^{\nu}} \cdots$$

in which the working volume Ω is subdivided, in a standard way, into sufficiently small local cells $\Omega(\mathbf{r})$ that are still large compared to the particle volume n^{-1}. (The unit cell volume in reciprocal space becomes $\Omega(\mathbf{r})^{-1}$.) The equilibrium fluctuation $\Delta f_{\alpha}^{\mathrm{eq}}$ is determined from standard statistical mechanics: [4]

$$\Delta f_\alpha^{\mathrm{eq}} \equiv k_{\mathrm{B}} T \frac{\partial f_\alpha^{\mathrm{eq}}}{\partial \varepsilon_{\mathrm{F}}(\mathbf{r})} = f_\alpha^{\mathrm{eq}}(1 - f_\alpha^{\mathrm{eq}}).$$

Here the local electrochemical potential $\varepsilon_{\mathrm{F}}(\mathbf{r}) = \mu - U_0(\mathbf{r})$, basically the Fermi level of the local population, is given by the global chemical potential μ offset by the mean-field (Hartree) potential $U_0(\mathbf{r})$.

Equation (3) is a rigorous, nonequilibrium, kinetic-theoretical relation [26]. It controls the physics of thermal fluctuations at length scales greater than the metallic Fermi wavelength, which is itself short (0.2–10 nm) compared to mesoscopic device sizes (say 50–1000 nm).

2.5 Temperature Scaling

Two outcomes flow from Eq. (3). The first is that even the *nonequilibrium* thermal fluctuations in a degenerate conductor necessarily scale with the thermal energy $k_{\mathrm{B}} T$, whatever the value of the driving voltage. For a specific illustration, see Fig. 3. The closed microscopic form of the distribution $\Delta f_\alpha(t)$ is given explicitly in Sec. 3 below. For the moment we state a milder result, the sum rule for the total fluctuation strength in the degenerate limit:

$$\sum_\alpha \Delta f_\alpha(t) = \sum_\alpha \Delta f_\alpha^{\mathrm{eq}} \to k_{\mathrm{B}} T \sum_{\mathbf{r}} \Omega(\mathbf{r}) \mathcal{D}[\varepsilon_{\mathrm{F}}(\mathbf{r})], \quad (4\mathrm{a})$$

in which the Fermi-Dirac form of Δf^{eq} is used to introduce the density of states $\mathcal{D}$:

$$\frac{2}{\Omega(\mathbf{r})} \sum_{\mathbf{k}} \Delta f_\alpha^{\mathrm{eq}} = 2 k_{\mathrm{B}} T \sum_{\mathbf{k}} \left\{ \frac{\delta(\varepsilon_\alpha - \varepsilon_{\mathrm{F}}(\mathbf{r}))}{\Omega(\mathbf{r})} \right\}$$

$$\to k_{\mathrm{B}} T \mathcal{D}[\varepsilon_{\mathrm{F}}(\mathbf{r})] \qquad (4\mathrm{b})$$

where ε_α is the local band energy of a carrier.

There is an immediate corollary for the *current autocorrelation function*, which shapes the observable noise spectrum for the structure. The thermal current correlations will scale with Δf. Equation (4) asserts that the thermal contribution to noise must exhibit a strict proportionality to the base temperature T, even well away from the linear low-field regime (where the Johnson-Nyquist formula itself [34] enforces T-scaling).

A question arises naturally: How can this behavior be reconciled with the appearance of shot noise, a thermally insensitive effect? The kinetic-theoretical answer (which we justify, fully and formally, in Sec. 3) is uncompromising:

- *There is no continuous transformation (crossover) of thermal noise into shot noise.*

As a purely nonthermal fluctuation effect, shot noise can never satisfy the rigid sum rule expressed in Eqs. (3) and (4). Nor does it satisfy the fluctuation-dissipation theorem; an in-depth analysis of this and other essential distinctions between shot noise and thermal noise is given by Gillespie [24].

Equation (4), and Fig. 3, directly countermand the Landauer-Büttiker account of shot noise as all of one piece with thermal noise. [14]; thus they make a nontrivial statement. For a kinetic-equation approach to shot noise see Refs. [27,30]. For complete technical details of that approach, see Ref. [39].

$$\frac{\Delta f_{\mathbf{k}}^{\mathrm{eq}}}{k_{\mathrm{B}} T} = -\lim_{\mathbf{q} \to 0} \left[\lim_{\omega \to 0} \left(\frac{f_{\mathbf{k} - \mathbf{q}/2}^{\mathrm{eq}} - f_{\mathbf{k} + \mathbf{q}/2}^{\mathrm{eq}}}{\hbar \omega - \varepsilon_{\mathbf{k} + \mathbf{q}/2} + \varepsilon_{\mathbf{k} - \mathbf{q}/2}} \right) \right]$$

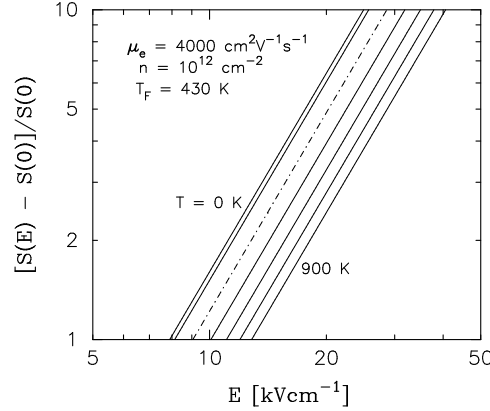for particle band energy $\varepsilon_{\mathbf{k}}$.

FIG. 3. Temperature scaling of degenerate hot-electron noise. The nonequilibrium excess spectrum is for carriers confined in an AlGaAs/InGaAs/GaAs heterojunction quantum well at electron density 10^{12} cm^{-2} and mobility 4000 cm^{2}V^{-1}s^{-1}. The hot-electron noise is plotted for fixed temperature (T goes from 0 to 900 K in increments of 150 K), as a function of applied electric field. Normalization is to the Johnson-Nyquist value $S(E=0) = 4Gk_{\mathrm{B}}T$. In the limit $T \to 0$ we have $S(E) \propto T/T_{\mathrm{F}}$ for Fermi temperature $T_{\mathrm{F}} = \varepsilon_{\mathrm{F}}/k_{\mathrm{B}}$. Degeneracy forces the hot-electron noise to vanish with temperature, so the ratio $[S(E) - S(0)]/S(0)$ is independent of T. In the limit $T \gg T_{\mathrm{F}}$ the electrons are classical. The excess noise is independent of T so that $[S(E) - S(0)]/S(0) \ll 1$. The dot-dashed line is for $T = 300$ K.

2.6 Noise Suppression via Degeneracy and Inhomogeneity

The second outcome of the microscopic theory leading to Eq. (3), as detailed in the next Section, is that a mesoscopic conductor which is strongly nonuniform manifests *Coulomb suppression of charge fluctuations* below those of a uniform reference medium, with otherwise identical transport characteristics [26].

One other condition is essential for Coulomb suppression: carrier degeneracy. Suppression is a unique effect of Fermi statistics, acting in conjunction with spatial inhomogeneity and Coulomb screening. It is not seen in a classical electron gas, where Maxwell-Boltzmann statistics leads to equipartition of the internal energy [34].

The mechanism of suppression is as follows. Degenerate carriers in a nonuniform channel experience some degree of localization. They will lower their total energy by a partial rearrangement, setting up a self-consistent field to screen their large charging energy, due to degeneracy and confinement. The confining potential can be engineered by spatially dependent doping, discontinuities in the band structure, or a combination of both, as in most III-V heterojunction quantum channels [38].

Taking the latter as our example, let us look for the effect of the large self-consistent Coulomb energy on the fluctuations of the two-dimensional electron gas (2DEG). A channel with density n_s in the plane of confinement contains Ωn_s carriers in area Ω:

$$N \equiv \Omega n_s = \Omega \mathcal{D} k_{\mathrm{B}} T \ln\left\{1 + \exp[(\mu - \varepsilon_0(n_s))/k_{\mathrm{B}}T]\right\}. \quad (5)$$

Here $\mathcal{D} = m^*/\pi\hbar^2$ is the 2DEG density of states. For simplicity we assume ground-state occupation only, at subband energy $\varepsilon_0(n_s)$. The density dependence of $\varepsilon_0(n_s)$ reflects the strong Coulomb repulsion within the 2DEG, confined in the quantum well perpendicular to the channel.

Equation (5) can be varied in two ways to arrive at the charge-fluctuation strength over the channel. If the internal potential is frozen, $\varepsilon_0(n_s)$ remains at a *fixed value*. With this variational restriction, the 2DEG form of Eq. (3) for the driven channel becomes [26]

$$\sum_{\alpha} \Delta f_{\alpha}(t) = \Delta N = k_{\mathrm{B}} T \frac{\partial N}{\partial \mu}\bigg|_{\varepsilon_0(n_s)}$$
$$= \frac{\Omega \mathcal{D} k_{\mathrm{B}} T}{1 + \exp[(\varepsilon_0(n_s) - \mu)/k_{\mathrm{B}}T]}. \quad (6)$$

Lifting the restriction on the internal potential now allows for the natural, self-consistent relaxation of the local field due to the charge fluctuations. We do this by including the negative-feedback term that comes from the density dependence of $\varepsilon_0(n_s(\mu))$, present on the right-hand side of Eq. (5). The self-screening of Δf_{α} then means that

$$k_{\mathrm{B}} T \frac{\delta N}{\delta \mu} = \sum_{\alpha} \widetilde{\Delta} f_{\alpha}^{\mathrm{eq}} = \left(1 - \frac{\delta \varepsilon_0}{\delta \mu}\right) \sum_{\alpha} \Delta f_{\alpha}^{\mathrm{eq}}$$
$$= \left(1 - \frac{1}{\Omega} \frac{\delta N}{\delta \mu} \frac{d\varepsilon_0}{dn_s}\right) k_{\mathrm{B}} T \frac{\partial N}{\partial \mu}\bigg|_{\varepsilon_0(n_s)} \quad (7a)$$

where $\widetilde{\Delta} f_{\alpha}^{\mathrm{eq}}$ is the equilibrium distribution of fluctuations, in the full presence of self-consistency. Eq. (7a)

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can be rearranged to give a closed expression for the total number fluctuation

$$\widetilde{\Delta} N = k_{\mathrm{B}} T \frac{\delta N}{\delta \mu} = \frac{\Delta N}{1 + \left(\dfrac{\Delta N}{\Omega k_{\mathrm{B}} T} \right) \dfrac{d\varepsilon_0}{dn_s}}, \qquad (7b)$$

in complete analogy with the Thomas-Fermi screening formula for the bulk electron gas [3].

Through the global-neutrality condition, Gauss' theorem again leads straight to a dynamical sum rule for the 2DEG fluctuations:

$$\sum_\alpha \widetilde{\Delta} f_\alpha(t) = \widetilde{\Delta} N = \frac{\Delta N}{1 + \left(\dfrac{\Delta N}{\Omega k_{\mathrm{B}} T} \right) \dfrac{d\varepsilon_0}{dn_s}}, \qquad (7c)$$

where $\widetilde{\Delta} f_\alpha(t)$ denotes the time-dependent mean-square distribution of the fluctuations out of equilibrium, with full self-consistency. Eq. (7c), like Eq. (3) before it, is an exact relation with a rigorous kinetic-theoretical basis [26].
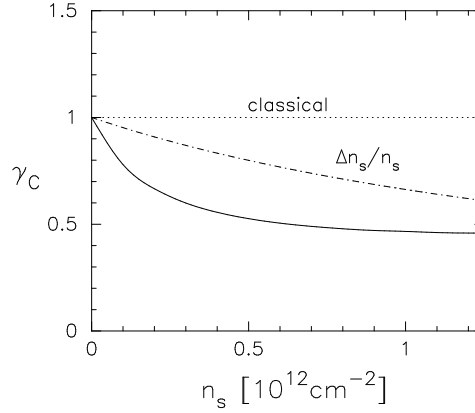

FIG. 4. Effect of inhomogeneous Coulomb screening on the quantum-well confined electron population in an AlGaAs/InGaAs/GaAs heterojunction, as a function of sheet electron density n_s. Solid line: the suppression coefficient for degenerate carrier fluctuations, $\gamma_{\mathrm{C}} \equiv \widetilde{\Delta} N / \Delta N$; refer to Eq. (7c) in the text. Dot-dashed line: The unscreened (free-carrier) ratio $\Delta N/N$ of mean-square number fluctuations to mean carrier number. This ratio measures the degeneracy of the system; a smaller value means higher degeneracy. Both $\Delta N/N$ and γ_{C} are intimately related to the system's compressibility; see Eqs. (9) and (10). Dotted line: in the classical limit both ratios are unity. *When there is no degeneracy, there is no inhomogeneous Coulomb suppression of the compressibility.*

In Figure 4 we show the behavior of equilibrium fluctuations in a pseudomorphic AlGaAs/InGaAs/GaAs heterojunction at room temperature. Under normal operating conditions, even without cryogenic cooling, the quantum confined electron gas suppresses its thermal fluctuations by up to 50% below the free-electron value (Eq. (6)).

Just as Eq. (3) *necessarily* enforces the temperature scaling of all nonequilibrium thermal fluctuations, so must Eq. (7) enforce, in an inhomogeneous mesoscopic contact, the scaling of nonequilibrium fluctuations with Coulomb suppression. Much more than that, *Coulomb suppression is completely determined by the equilibrium state.* This has definite – and observable – physical consequences.

We have previewed some of the major, and completely generic, results of the kinetic approach to mesoscopic transport. In particular, we have highlighted the *microscopic* structure of the fluctuations, and of their sum rules, as being vital to the makeup of basic nonequilibrium processes. We now discuss the technicalities of how this comes about.

3. NONEQUILIBRIUM KINETICS

The focus of this section is on the conceptual structure of the formalism, with mathematics in support. First we recapitulate the open-system assumptions previewed in Sec. 2. We link these to the essential *sum rules* that the fluctuations of an electron gas must satisfy. Then we show that transmissive-diffusive models are in violation of at least one of these constraints: the compressibility sum rule. Finally, we survey our rigorous kinetic solution for transport and noise.

Together with every other model of current and noise in metals, including the transmissive-diffusive description [9–14], our kinetic approach requires

- an ideal thermal bath regulating the size of energy exchanges with the conductor, while itself always remaining in the equilibrium state;

- ideal macroscopic carrier reservoirs (leads) in open contact with the conductor, without themselves being driven out of their local equilibrium;

- absolute charge neutrality of the leads, and overall neutrality of the intervening conductor.

This standard scheme, consistently applied within the standard framework of Boltzmann and, later, of Landau and Silin [3,4,15], puts specific and tight constraints on the behavior of nonequilibrium current noise.

The electron gas in each asymptotic lead is unconditionally neutral, and satisfies canonical identities for *compressibility* and *perfect screening* [2,3]. It has long been understood that they embody the quantitative effects of degeneracy (compressibility sum rule) and of Gauss' theorem (perfect-screening sum rule).

Each criterion entails a precise numerical relation between the mean charge *density* and its *fluctuation*. Some feeling for the cardinal role of the electron-gas sum rules, in noise and transport together, can be gained by looking more closely at the compressibility.

3.1 Compressibility: a Case Study in Sum Rules

3.1.1 Compressibility and Electron-Gas Physics

The compressibility sum rule links the local physical density of the electron gas $n(\varepsilon_{\mathrm{F}}(\mathbf{r}))$ to the system's local, screened polarization function $\chi_0(q \ll k_{\mathrm{F}}, \omega = 0)$ in its adiabatic limit, for wavelengths long relative to the inverse Fermi wavevector k_{F}^{-1}. Thus [3]

$$\kappa \equiv \frac{1}{n^2}\frac{\partial n}{\partial \varepsilon_{\mathrm{F}}} = -\frac{1}{n^2}\chi_0(0,0) \equiv \frac{2}{n^2\Omega(\mathbf{r})}\sum_{\mathbf{k}}\frac{\Delta f_\alpha^{\mathrm{eq}}}{k_{\mathrm{B}}T}. \quad (8)$$

Comparison with Eq. (3) immediately shows the intimate connection between this canonical equilibrium relation, and the conservation of total fluctuation strength in a conductor taken *out* of equilibrium.

Let us go to the global form of the compressibility rule,

$$\frac{\Omega}{N^2}\frac{\partial N}{\partial \mu} = \frac{\Omega}{N^2 k_{\mathrm{B}}T}\sum_{\mathbf{r}}\Omega(\mathbf{r})\langle\Delta f^{\mathrm{eq}}(\mathbf{r})\rangle$$
$$= \frac{\Omega}{Nk_{\mathrm{B}}T}\frac{\Delta N}{N}, \quad (9)$$

where the trace over spin and momentum states is $\langle\cdots\rangle \equiv 2/\Omega(\mathbf{r})\sum_{\mathbf{k}}\cdots$. We make three observations.

- In the limit of the classical gas, $\Delta N = N$. Then the ideal-gas law shows that the right-hand side is the inverse of the pressure. The pressure is the thermodynamic bulk modulus, κ^{-1}.

  In the quantum regime, $\Delta N < N$. The exclusion principle keeps the electrons apart, making the system stiffer so that (in a loose sense) this is the Fermi-gas analog of van der Waals' hard-core model.

- Electron-hole symmetry is fundamental. The microscopic basis of compressibility lies within the same *electron-hole pair fluctuations* that determine the structure of the polarization response function $\chi_0(q, \omega)$; see also Footnote 4, Section 2.4 above. The dynamical evolution of the electron-hole pair excitations within $\chi_0(q, \omega)$ is *kinematically correlated* by microscopic charge and current conservation [5] expressed through the electron-hole symmetry of transport [2,3]; refer also to Fig. 1(b). The

---

[5]Current conservation is frequently discussed in the sense of an augmented particle flux that includes the *displacement* term associated with Poisson's equation. The sum of the two has zero divergence; consider the equation of conservation (continuity)

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{J} = 0,$$

which comes from taking traces over $\mathbf{k}$ in the equation of motion (refer to Eq. (16) in the text). Poisson's equation for the density gives

$$-4\pi e\frac{\partial n}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\partial}{\partial \mathbf{r}}\cdot\epsilon\mathbf{E}\right).$$

Then the continuity equation can be recast as

very same, inherently correlated, electron-hole processes determine the noise [25,26].

- For a nonuniform conductor, we must compute the total response to a change in global chemical potential. As before (recall Eq. (7c)) we now have

$$
\begin{aligned}
\frac{\Omega}{N^2}\frac{\delta N}{\delta\mu} &= \frac{\Omega}{N^2 k_{\mathrm{B}}T} \\
&\times \sum_{\mathbf{r}} \Omega(\mathbf{r}) \frac{\langle \Delta f^{\mathrm{eq}}(\mathbf{r})\rangle}{1 + \dfrac{\langle \Delta f^{\mathrm{eq}}(\mathbf{r})\rangle}{k_{\mathrm{B}}T}\dfrac{dU_0(\mathbf{r})}{dn(\mathbf{r})}} \\
&\equiv \frac{\Omega}{Nk_{\mathrm{B}}T}\frac{\widetilde{\Delta N}}{N}.
\end{aligned}
\tag{10}
$$

The internal Coulomb correlations, which determine the local mean-field potential $U_0(\mathbf{r})$, increase the free energy of the electrons. This makes the electrons stiffer yet, over and above the exchange correlations evident in Eq. (9). It is a classic illustration of *Coulomb screening* at work, and is obviously a major physical process in mesoscopic structures whose spatial irregularities are large, or else approach the scale of the screening length [26].

3.1.2 Compressibility and Transmissive-Diffusive Phenomenology

In Section 2 we discussed how the total fluctuation strength of a mesoscopic conductor is invariant, whether it is in equilibrium or not. We now see that this is closely tied to the microscopics of the compressibility. One should therefore ask for the corresponding behavior of ΔN in a typical transmissive-diffusive model.

As a concrete example we take the noise theory of Martin and Landauer [8] for an electronic conductor. (We could as well have taken the Landauer-Büttiker description [7,14].) In addition, we recall that de Jong and Beenakker have argued for an equivalence between the transmissive-diffusive method and that of semiclassical (Boltzmann-Langevin) theory [13].

The model of Ref. [8] builds up the current-current correlation function from the set of all possible quantum-transmission events through the conducting region. We take their one-dimensional (1D) noise calculation for a sample of length L and (constant) transmission probability $\mathcal{T}$. Following their Eqs. (2.6)–(2.15), the linear number fluctuation δN can be computed. [6] We arrive at the mean-square value

$$
\begin{aligned}
\Delta N \equiv \langle(\delta N)^2\rangle - \langle(\delta N)\rangle^2\rangle &= L\frac{n}{2\varepsilon_{\mathrm{F}}}\left[\mathcal{T}^2 k_{\mathrm{B}}T + \mathcal{T}(1-\mathcal{T})\frac{\mu_{\mathrm{S}}-\mu_{\mathrm{D}}}{2}\coth\left(\frac{\mu_{\mathrm{S}}-\mu_{\mathrm{D}}}{2k_{\mathrm{B}}T}\right)\right] \\
&= N\frac{k_{\mathrm{B}}T}{2\varepsilon_{\mathrm{F}}}\left[\mathcal{T} + \frac{\mathcal{T}(1-\mathcal{T})}{3}\left(\frac{\mu_{\mathrm{S}}-\mu_{\mathrm{D}}}{2k_{\mathrm{B}}T}\right)^2 + \mathcal{O}\left(((\mu_{\mathrm{S}}-\mu_{\mathrm{D}})/2k_{\mathrm{B}}T)^4\right)\right],
\end{aligned}
\tag{11}
$$

where $n = 2k_{\mathrm{F}}/\pi$ is the 1D carrier density while μ_{S} and μ_{D} are, respectively, the "chemical potentials" assumed for the equilibrium state of the source and drain leads.

Typically, as does *every other* diffusive model, the Martin-Landauer theory supposes that the EMF potential eV fixes the difference between the source and drain chemical potentials:

$$
\mu_{\mathrm{S}} - \mu_{\mathrm{D}} \equiv eV.
\tag{12}
$$

It follows directly that, to leading order in the EMF, the diffusively driven Martin-Landauer theory predicts

$$
\frac{\Delta N}{N} = \left[\frac{\Delta N}{N}\right]^{\mathrm{eq}}\left[\mathcal{T} + \frac{\mathcal{T}(1-\mathcal{T})}{3}\left(\frac{eV}{2k_{\mathrm{B}}T}\right)^2\right],
\tag{13}
$$

in which

$$
\left[\frac{\Delta N}{N}\right]^{\mathrm{eq}} = \frac{k_{\mathrm{B}}T}{2\varepsilon_{\mathrm{F}}}
$$

$$
\frac{\partial}{\partial\mathbf{r}}\cdot\left[\mathbf{J} + \frac{\partial}{\partial t}\left(-\frac{\epsilon}{4\pi e}\mathbf{E}\right)\right] = 0.
$$

The total flux is divergenceless *if and only if* the originating equation of motion is gauge invariant. There is no way to guarantee this result otherwise.

[6]The quantity δN has nothing to do with the electron-hole pair fluctuations intrinsic to the system. It is generated purely by the carriers that enter and leave the device, in Poissonian fashion. As it turns out, δN is simply proportional to the current fluctuation for that model. Note that this is characteristic of *all* models built with the same transmissive-diffusive arguments as Martin and Landauer's.

is the 1D equilibrium ratio of the total mean-square number fluctuation to total carrier number.

If Eqs. (2) and (3) are correct, as we will prove, then Eq. (13) violates the compressibility sum rule. Therefore the fluctuation structure of this diffusive model also violates number conservation.

This is the cost of neglecting electron-hole symmetry in the construction of pseudodiffusive transport. All transmissive-diffusive models do this without exception. For an interesting comment on such violations, see Ref. [14], Eq. (51) and subsequent paragraph.

One can now answer the two core questions posed in our Introduction:

- **Q**. Do transmissive-diffusive theories fully respect *all* of the essential physics of the electron gas?
  A. No.

- **Q**. If not, *why* not?
  A. There are two reasons.

  (i) The total fluctuation ΔN in the transmissive-diffusive models depends on the transport parameter $\mathcal{T}$. It vanishes with $\mathcal{T}$. As we have seen, the compressibility is an equilibrium property insensitive to external sources of elastic scattering (such as potential barriers) which fix $\mathcal{T}$. Thus Eq. (13) cannot recover the physical compressibility, *even in the elementary zero-field limit of such models.* Nor is it possible to invoke Coulomb suppression to account for the spurious dependence on $\mathcal{T}$. This unphysical result is for a uniform, free-electron model.

  (ii) Such theories grossly mistreat the role of the equilibrium state in each bounding reservoir. The relevant thermodynamic chemical potentials are not at all μ_S and μ_D, but the *undisturbed* equilibrium values. These remain *locally invariant* within each lead. Only then can the electron reservoirs fulfill their role: to stabilize, screen, and confine the nonequilibrium fields and their fluctuations within the active region [18,19,25,29]. (At zero current, of course, each lead chemical potential aligns with the global μ.)

  In view of the prevalence of pseudodiffusive thinking, one cannot reassert sufficiently strongly the overwhelming physical importance of this unconditional constraint: the reservoirs' chemical potentials are *always local* and *always undisturbed.*

  Unequivocally, these *local-equilibrium* quantities are the only ones that can appear in the transport description. That is the only rule compatible with the microscopic structure of the electron gas, both in the sample and its stabilizing leads.

3.2 Nonequilibrium Carrier Distribution

To confirm the fluctuation sum rules Eqs. (3) and (7c), disconfirming in the process the counterfeit fluctuation equation (11), we must show that the nonequilibrium carrier fluctuations are *linear functionals* of the equilibrium ones. From this follow all of the results that we have already discussed.

We will need the one-electron equilibrium distribution. It is

$$f_\alpha^{\mathrm{eq}} = \left[1 + \exp\left(\frac{\varepsilon_\mathbf{k} + U_0(\mathbf{r}) - \mu}{k_B T} \right) \right]^{-1}. \qquad (14)$$

The conduction-band energy $\varepsilon_\mathbf{k}$ can vary (implicitly) with $\mathbf{r}$ if the local band structure varies, as in a heterojunction. The mean-field potential $U_0(\mathbf{r})$ vanishes asymptotically in the leads, and satisfies the self-consistent Poisson equation (ϵ is the background-lattice dielectric constant)

$$\nabla^2 U_0 \equiv e \frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{E}_0 = -\frac{4\pi e^2}{\epsilon} \Big(\langle f^{\mathrm{eq}}(\mathbf{r}) \rangle - n^+(\mathbf{r}) \Big) \qquad (15)$$

in which, for later use, $\mathbf{E}_0(\mathbf{r})$ is the internal field in equilibrium (recall that a nonuniform system sustains nonzero internal fields). The (nonuniform) neutralizing background density $n^+(\mathbf{r})$ goes to the same constant value, n, as the electrons in the (uniform) leads.

We study the semiclassical Boltzmann–Landau-Silin equation. There is a substantial body of work, at every level, on this transport equation. Among the analyses that we have found most useful, we cite Refs. [20,40,41] for Boltzmann-oriented kinetic descriptions and Refs. [3,15,42] for more Fermi-liquid-oriented ones in the spirit of Landau and Silin.

The kinetic equation, subject to the total internal field $\mathbf{E}(\mathbf{r}, t)$, can be written as

$$\left(\frac{\partial}{\partial t} + D_\alpha[\mathbf{E}(\mathbf{r}, t)] \right) f_\alpha(t) = -\mathcal{W}_\alpha[f]. \qquad (16)$$

Here $D_\alpha[\mathbf{E}] \equiv \mathbf{v_k} \cdot \partial/\partial \mathbf{r} - (e\mathbf{E}/\hbar) \cdot \partial/\partial \mathbf{k}$ is the convective operator and $\mathcal{W}_\alpha[f]$ is the collision operator, whose kernel (local in real space) is assumed to satisfy detailed balance, as usual [20]. Even for single-particle impurity scattering, Pauli blocking of the outgoing scattering states still means that $\mathcal{W}$ is generally nonlinear in the nonequilibrium function $f(t)$.

Since we follow the standard Boltzmann–Landau-Silin formalism [3,20,41], all of our results will comply with the conservation laws. The nonlinear properties of these results extend as far as the inbuilt limits of the semiclassical framework. These go much further than any model restricted to the weak-field domain. Since we rely expressly on the whole fluctuation structure provided by Fermi-liquid theory [3], all of the fundamental sum rules are incorporated.

We develop our theory for the steady-state distribution f_α out of equilibrium by expressing it as an explicit functional of the equilibrium distribution. The latter satisfies

$$D_\alpha[\mathbf{E}_0(\mathbf{r})]f_\alpha^{\text{eq}} = 0 = -\mathcal{W}_\alpha[f^{\text{eq}}], \qquad (17)$$

the second equality following by detailed balance. Subtract the corresponding sides of Eq. (17) from both sides of the time-independent version of Eq. (16). On introducing the difference function $g_\alpha \equiv f_\alpha - f_\alpha^{\text{eq}}$, one obtains

$$\sum_\beta \left(\mathcal{I}_{\alpha\beta} D_\beta[\mathbf{E}(\mathbf{r}_\beta)] + \mathcal{W}'_{\alpha\beta}[f] \right) g_\beta$$
$$= \frac{e[\mathbf{E}(\mathbf{r}) - \mathbf{E}_0(\mathbf{r})]}{\hbar} \cdot \frac{\partial f_\alpha^{\text{eq}}}{\partial \mathbf{k}} - \mathcal{W}''_\alpha[g]. \qquad (18)$$

The unit operator in Eq. (18) is

$$\mathcal{I}_{\alpha\alpha'} \equiv \left[\frac{\delta_{\mathbf{k}\mathbf{k}'}}{\Omega(\mathbf{r})} \right] [\Omega(\mathbf{r})\delta_{\mathbf{r}\mathbf{r}'}]$$

and the linearized operator $\mathcal{W}'[f]$ is the variational derivative

$$\mathcal{W}'_{\alpha\alpha'}[f] \equiv \frac{\delta \mathcal{W}_\alpha[f]}{\delta f_{\alpha'}}.$$

Last, the collision term

$$\mathcal{W}''_\alpha[g] \equiv \mathcal{W}_\alpha[f] - \mathcal{W}_\alpha[f^{\text{eq}}] - \sum_\beta \mathcal{W}'_{\alpha\beta}[f]g_\beta$$

carries the residual nonlinear contributions. Although $\mathcal{W}_\alpha[f^{\text{eq}}]$ is identically zero by detailed balance, $\mathcal{W}'_{\alpha\alpha'}[f^{\text{eq}}]$ is not. We must formally keep the equilibrium quantity, via $\mathcal{W}''_\alpha[g]$, on the right-hand side of Eq. (18) because we will require its variational derivative.

Global neutrality enforces the fundamental constraint

$$\sum_\alpha g_\alpha = \sum_\mathbf{r} \Omega(\mathbf{r})\langle g(\mathbf{r})\rangle = 0. \qquad (19)$$

We need not elaborate; Eq. (19) is the immediate consequence of the general boundary conditions introduced at the Section's beginning. From it, all of the sum-rule results are derived.

The leading right-hand term in Eq. (18) is responsible for the functional dependence of g on the equilibrium distribution (this is important because dependence on *equilibrium-state* properties carries through to the variationally derived steady-state fluctuations). The electric-field factor can be written as

$$\mathbf{E}(\mathbf{r}) - \mathbf{E}_0(\mathbf{r}) \equiv \widetilde{\mathbf{E}}(\mathbf{r}) = \mathbf{E}_{\text{ext}}(\mathbf{r}) + \mathbf{E}_{\text{ind}}(\mathbf{r}),$$

where $\mathbf{E}_{\text{ext}}(\mathbf{r})$ is the external driving field, and the induced field $\mathbf{E}_{\text{ind}}(\mathbf{r})$ obeys

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{E}_{\text{ind}} = -\frac{4\pi e}{\epsilon} \langle g(\mathbf{r})\rangle. \qquad (20)$$

Equation (20) guarantees that g vanishes in the equilibrium limit. This maintains the so-called adiabatic connection of the nonequilibrium solution f to f^{eq}.

3.3 Nonequilibrium Fluctuations; Analytical Form

Now we consider the nonequilibrium fluctuation $\Delta f_\alpha(t)$. It satisfies the (well documented) linearized equation of motion [40,41]

$$\sum_\beta \left[\mathcal{I}_{\alpha\beta} \left(\frac{\partial}{\partial t} + D_\beta[\mathbf{E}(\mathbf{r}_\beta)] \right) + \mathcal{W}'_{\alpha\beta}[f] \right] \Delta f_\beta(t) = 0. \quad (21)$$

This equation remains subject to the same unconditional boundary constraints that we have discussed. In the Landau Fermi-liquid regime, it generates all of the dynamical properties of the fluctuating electron gas. Once it is solved, *all* of the physical properties of the current fluctuations can be computed.

For the adiabatic $t \to \infty$ limit, $\Delta f_\alpha(t) \to \Delta f_\alpha$ represents the average strength of the spontaneous background fluctuations, induced in the steady state by the ideal thermal bath. It is one of two essential components that determine the dynamical fluctuations. The other component is the dynamical Green function for the inhomogeneous version of Eq. (21). See Ref. [26].

In a strongly degenerate system Δf_α dictates the explicit T-scaling of all thermally based noise through its functional dependence on the equilibrium distribution $\Delta f_\mathbf{k}^{\text{eq}}(\mathbf{r})$. We saw this in Eqs. (4) and (7). Now we prove it.

Define the variational derivative

$$G_{\alpha\alpha'}[f] \equiv \left. \frac{\delta g_\alpha}{\delta f_{\alpha'}^{\text{eq}}} \right|_\mathbf{E}. \qquad (22)$$

This operator obeys a steady-state equation obtained from Eq. (18) by taking variations on both sides. Note that we restrict the variation by keeping the total internal field constant. This provides us with the nonequilibrium Fermi-liquid response of the system (dominated by degeneracy). The self-consistent Coulomb field fluctuations can be obtained, systematically, by lifting the variational restriction. See our Ref. [26].

The equation for G is

$$\sum_\beta \left(\mathcal{I}_{\alpha\beta} \mathcal{D}_\beta[\mathbf{E}(\mathbf{r}_\beta)] + \mathcal{W}'_{\alpha\beta}[f] \right) G_{\beta\alpha'} = \mathcal{I}_{\alpha\alpha'} \frac{e\widetilde{\mathbf{E}}(\mathbf{r}')}{\hbar} \cdot \frac{\partial}{\partial \mathbf{k}'} - \mathcal{W}'_{\alpha\alpha'}[f] + \mathcal{W}'_{\alpha\alpha'}[f^{eq}]. \qquad (23)$$

The explicit and closed form for G, which we do not give here, is obtained from knowledge the dynamical Green function for the linearized equation of motion, Eq. (21) [25,26]. The main point, of utmost physical importance, is that the expression

$$\Delta f_\alpha = \Delta f_\alpha^{\text{eq}} + \sum_{\alpha'} G_{\alpha\alpha'} \Delta f_{\alpha'}^{\text{eq}} \qquad (24)$$

satisfies the steady-state form of Eq. (21) *exactly*. In the form above, Δf_α is the definitive solution for the steady-state, mean-square fluctuation in nonequilibrium transport.

Eqs. (3) and (4) can now be confirmed in steady state by invoking the unconditional neutrality of g; see Eq. (19). This immediately implies

$$\sum_\alpha G_{\alpha\alpha'} = 0 \quad \text{for all } \alpha'. \qquad (25a)$$

Hence

$$\sum_\alpha \Delta f_\alpha = \sum_\alpha \Delta f_\alpha^{\text{eq}} + \sum_{\alpha'} \left(\sum_\alpha G_{\alpha\alpha'} \right) \Delta f_{\alpha'}^{\text{eq}}$$
$$= \sum_\alpha \Delta f_\alpha^{\text{eq}}, \qquad (25b)$$

which establishes the static form of the compressibility sum rule; an *exact* constraint on the nonequilibrium carrier fluctuations in a mesoscopic conductor. It holds under very general boundary conditions and modes of scattering (quasiparticle interactions are included in the collision integral $\mathcal{W}[f]$, as well as external collision processes). A well controlled theory of mesoscopic noise *must take the compressibility sum rule into account* at the very least (there are several others [3]).

The stationary fluctuation properties of a driven system are intimately connected to its dynamic response. We end this technical discussion with a description of the noise spectral density.

3.4 Nonequilibrium Fluctuations: Dynamics

3.4.1 Dynamic Fluctuation Structure

The time-dependent Green function is the variational derivative (with Coulomb effects restricted)

$$R_{\alpha\alpha'}(t - t') \equiv \theta(t - t') \left. \frac{\delta f_\alpha(t)}{\delta f_\alpha(t')} \right|_{\mathbf{E}}; \qquad (26)$$

$\theta(t - t')$ is the Heaviside unit-step function. In the low-field limit, the Fourier transform of R is closely related to the internal makeup of the dynamic polarization $\chi_0(q, \omega)$ [3]. It can be solved routinely [21,22,40,41].

The exact solution to the equation of motion for the dynamical fluctuation, Eq. (21), is

$$\Delta f_\alpha(t) = \sum_{\alpha'} R_{\alpha\alpha'}(t) \Delta f_{\alpha'}. \qquad (27a)$$

The conserving nature of R implies that $\sum_\alpha R_{\alpha\alpha'}(t) = 1$ for all α'. It follows that [25]

$$\sum_\alpha \Delta f_\alpha(t) = \sum_{\alpha'} \sum_\alpha R_{\alpha\alpha'}(t) \Delta f_{\alpha'} = \sum_{\alpha'} \Delta f_{\alpha'}. \qquad (27b)$$

With Eq. (27b) and Eq. (25b) in association, we complete the promised derivation of Eq. (3), which essentially fixes the dynamic global compressibility in a mesoscopic conductor out of equilibrium.

From the point of view of microscopic analysis, our derivation is *entirely standard* and thus definitive. The only way to circumvent its negative implication for transmissive-diffusive theory, would be to show that its long-established basis in electron-gas physics – going back almost a century – is erroneous.

The proof for the exact Coulomb-suppressed compressibility Eq. (10) develops along parallel lines, apart from the added self-consistency feature. It is fully set out in Ref. [26].

3.4.2 Current-Current Correlation

For the current autocorrelation we require the *transient* part of the propagator R [40,41],

$$C_{\alpha\alpha'}(t) = R_{\alpha\alpha'}(t) - R_{\alpha\alpha'}(t \to \infty). \qquad (28)$$

The transient propagator carries all the dynamical correlations. As is standard practice [40,41], the flux autocorrelation can be written down directly in terms of C and Δf:

$$S_{JJ}(\mathbf{r}, \mathbf{r}'; t) \equiv \frac{2}{\Omega(\mathbf{r})\Omega(\mathbf{r}')} \sum_{\mathbf{k}} \sum_{\mathbf{k}'} [-e(v_x)_{\mathbf{k}}] C_{\alpha\alpha'}(t)$$
$$\times [-e(v_x)_{\mathbf{k}'}] \Delta f_{\alpha'}, \qquad (29)$$

where for illustration we select the x-components of the velocities. (This is the most relevant term for a uniform conductor with the driving field acting along the x-axis.)

Let us outline the physical meaning of Eq. (29). In steady state, the average fluctuation strength is Δf. Once a spontaneous thermal fluctuation (with this strength) arises within the system, it evolves and decays as a result of collisional processes. The transient evolution, and its characteristic time constant, are given by C. There are three parts to the exercise:

(a) the object $\mathbf{v}' \Delta f'$ represents, in the mean, a spontaneous flux fluctuation.

(b) After time t, the fluctuation has evolved to $C(t)\mathbf{v}'\Delta f'$.

(c) The velocity autocorrelation that describes this dynamical process is $\mathbf{v}C(t)\mathbf{v}'\Delta f'$.

3.4.3 Temperature Scaling

Since S_{JJ} scales with Δf, which itself scales with Δf^{eq}, our conclusion for the current-current fluctuation in a degenerate conductor is inescapable.

- *In a metallic system, the current-current correlator always scales with temperature T.*

This strict result leaves transmissive-diffusive models [14] in a difficult, indeed untenable, position. On the one hand, their current-current correlator must revert to the mandatory Johnson-Nyquist form at low fields. This is canonically proportional to T. On the other hand, consider the high-field, low-frequency limit of the noise spectral density in the theory of Ref. [8], whose form is identical for all of the theories in question:

$$\mathcal{S}(V;\omega=0)$$

$$= 4\frac{e^2\mathcal{T}}{\pi\hbar}\left[\mathcal{T}k_{\mathrm{B}}T + (1-\mathcal{T})\frac{\mu_{\mathrm{S}}-\mu_{\mathrm{D}}}{2}\coth\left(\frac{\mu_{\mathrm{S}}-\mu_{\mathrm{D}}}{2k_{\mathrm{B}}T}\right)\right]$$

$$\rightarrow 4\frac{e^2\mathcal{T}}{\pi\hbar}\left(\mathcal{T}k_{\mathrm{B}}T + (1-\mathcal{T})\frac{eV}{2}\right). \qquad (30)$$

The dominant term is the last one on the right-hand side, ascribed to shot-noise processes. It does not scale with temperature, as required by the compressibility sum rule. It follows that the current-current correlator in such a model, on which the derivation of $\mathcal{S}(V,\omega)$ is based, cannot be the canonical one, Eq. (29) [40,41]. Hence

- *Equation (30) and the Landauer-Büttiker-Imry phenomenology that leads directly to it, are in manifest and irreconcilable conflict with canonical microscopics.*

Does the strict T-scaling of S_{JJ} mean that shot noise is an ill-defined concept in the kinetic description of a degenerate mesoscopic conductor? Not at all. Shot noise is a real effect

The canonically obtained form for S_{JJ} – with its T-scaling – clearly implies that shot-noise fluctuations of a degenerate conductor must have a physical origin, and behavior, entirely distinct from its thermal fluctuations. Therefore

- *Shot noise must have a microscopic description entirely distinct from that for "hot-electron" noise, incorporated within Eq. (29).*

We do not give the kinetic-theoretical treatment of shot noise in the present review. Such a treatment is available in our Refs. [27] and [39]. In essence, shot noise is a time-of-flight process measured between the device boundaries. (Its intuitive meaning is well depicted by Martin and Landauer [8], though in a formalism incompatible with the electron gas.)

Shot noise involves discrete changes in the total carrier number N. By contrast, thermal noise is a volume-distributed process. It involves continuous changes of internal energy. The two are numerically very different, though both share the same variational, microscopic building blocks: C and $\partial f/\partial\mu$ or, in the case of shot noise, $\partial f/\partial N$.

3.5 Coda

Our primary goal is met. We have described the structure and physical consequences of a kinetic approach to noise that is strictly conserving. The intent of our first-principles mesoscopics program is aptly put by Imry and Landauer [9]:

Kubo's linear-response theory is essentially an extended theory of polarizability. Some supplementary hand-waving is needed to calculate a dissipative effect such as conductance, for a sample with boundaries where electrons enter and leave... After all, no theory that ignores the interfaces of a sample to the rest of its circuit can possibly calculate the resistance of such a sample of limited extent.

No more need be said, save for four incidental remarks.

- A properly constituted conductance and fluctuation theory of the electron gas **IS** a theory of the polarizability [25,29]. A polarization-based model is not a matter of taste; the physics of electron-hole processes in the electron gas [3] demands it. All self-styled alternatives are nonconservative. Furthermore, the Kubo conductance formula [37] emerges directly from an axiomatic derivation of the fluctuation-*dissipation* theorem (an accomplishment beyond Ref. [9] and its like).

- No hand-waving, supplementary *or otherwise*, is needed to calculate dissipation. That is automatic for a model (such as Kubo's) which guarantees its fluctuation-dissipation theorem from first principles [29], rather than having to take it on faith.

- It is not merely well known how to include dissipation; it is *obligatory* to do so explicitly, microscopically, and in perfect harmony with gauge invariance. Even the humble Drude model – with its supposedly "primitive" understanding – easily achieves that much, at least [18,29,43,44]. The same cannot be said of purely intuitive schemes.

- Transmissive-diffusive phenomenology *itself* ignores the avowedly crucial interface physics. That is why it mistreats the canonical compressibility so grossly.

Kinetic theory, unlike the pseudodiffusive mindset, respects the sum rules that have been established – universally and decades ago [2–4] – as definitive expressions of the Fermi-liquid origin of electron-hole correlations. They govern two phenomena, *conduction* and *noise*. It remains to give a major application of what is, in every way, a thoroughly conventional microscopic approach: the behavior of high-current thermal noise in one-dimensional ballistic wires and quantum point contacts.

4. BALLISTIC NOISE

We review our results for 1D ballistic noise, reported recently and more fully in Ref. [28]. That work has the complete details. The quantity that we wish to calculate is the long-time limit of the thermal-noise correlation

$$\mathcal{S}(V) \equiv 4 \int_0^\infty dt \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dx' \; \frac{S_{JJ}(x, x'; t)}{L^2} \quad (31)$$

for a 1D mesoscopic conductor of length L. Our calculation covers both diffusive and ballistic cases, but we focus on the latter.

4.1 Transport Problem

Recall Fig. 2 for a mesoscopic wire in close electrical contact with its reservoirs. The wire is *uniform*, except possibly in the restricted fringing regions where the current, as it is injected and extracted, strongly perturbs the local electrons. This induces a net charge displacement, responsible for Landauer's resistivity dipole [5], which is also the EMF. Under the conditions of strong screening and *phase breaking* imposed by the reservoirs, it can be argued that the carriers crossing the active region have no detailed memory of the boundary disturbances. Within the wire, they are Markovian and obey the spatially homogeneous form of the kinetic equation, Eq. (16).

Furthermore, the explicit presence of the current source and sink [17], with their associated regions of strong relaxation by scattering, means physically that the dissipative effects of *inelastic collisions* must be explicitly represented. Once again, we stress that vague appeals to dissipative relaxation in the leads' asymptotic equilibrium state [9] avail nothing to the description of real *driven* mesoscopic transport.

The ballistic kinetic equation is

$$\frac{\partial f_k}{\partial t} + \frac{eE}{\hbar} \frac{\partial f_k}{\partial k} = -\frac{1}{\tau_{\mathrm{in}}(\varepsilon_k)} \left(f_k(t) - \frac{\langle \tau_{\mathrm{in}}^{-1} f(t) \rangle}{\langle \tau_{\mathrm{in}}^{-1} f^{\mathrm{eq}} \rangle} f_k^{\mathrm{eq}} \right)$$
$$- \frac{1}{\tau_{\mathrm{el}}(\varepsilon_k)} \frac{f_k(t) - f_{-k}(t)}{2}. \quad (32)$$

The uniform driving field is $E = V/L$. For the collision operator we adopt a Boltzmann-Drude form that includes the inelastic collision time $\tau_{\mathrm{in}}(\varepsilon_k)$ as well as the elastic time $\tau_{\mathrm{el}}(\varepsilon_k)$. The structure of the inelastic collision contribution on the right-hand side automatically ensures charge and current conservation.

The solution to Eq. (32) can be written down analytically for collision times that are independent of particle energy [28]. In the sense of our open-system kinetics, the 1D wire is collision-free (that is, ballistic) when the dominant mean free paths $v_F \tau_{\mathrm{in}}$ and $v_F \tau_{\mathrm{el}}$ (for Fermi velocity v_F) are at their *maximum span*. That happens only when both are equal to the "ballistic length" L between the regions of strong relaxation, at the current entry and exit points. The ballistic length is therefore set by the *longest* mean free path in the problem, which cannot be greater than the distance between the sites for relaxation.

This ballistic condition leads straight to Landauer's ideal quantized conductance [28]:

$$G = \frac{I}{V} = \frac{e^2}{\pi \hbar} \quad (33)$$

for a single, occupied subband within the (open) 1D wire. When conditions are nonideal, so that the wire is either "elastic–diffusive" ($\tau_{\mathrm{el}} < \tau_{\mathrm{in}} = L/v_F$) or "inelastic–dissipative" ($\tau_{\mathrm{in}} < \tau_{\mathrm{el}} = L/v_F$), then

$$G = \frac{e^2}{\pi \hbar} \left(1 - \frac{|\tau_{\mathrm{in}} - \tau_{\mathrm{el}}|}{\tau_{\mathrm{in}} + \tau_{\mathrm{el}}} \right). \quad (34)$$

The second ratio on the right-hand side plays the role of the Landauer-Büttiker transmission probability $\mathcal{T}$ except that inelastic effects are fully included; the transmissive-diffusive treatment of $\mathcal{T}$ admits only coherent, purely elastic, scattering [9,14].

4.2 Ballistic Hot-electron Noise

Nonideality in the 1D conductance is well documented in many ballistic tests of Landauer's quantized formula. Nonideal conductance appears even in the most refined state-of-the-art measurements, notably the recent ones by de Picciotto *et al.* [45]. It is of great interest to predict the corresponding nonideal behavior of the nonequilibrium thermal noise.

Our conserving kinetic theory, worked out according to the methods described in Sec. 3, results in a noise spectral density that is exact for the transport model of Eq. (32). Expressed as the thermal hot-electron excess noise within a given subband of carrier states in the 1D conductor, say the ith one, it is

$$\mathcal{S}_i^{\mathrm{xs}}(V) = \mathcal{S}_i(V) - 4G_i k_{\mathrm{B}} T = \frac{\kappa_i}{\kappa_i^{\mathrm{cl}}} \frac{2e^2 I^2}{G_i m^* L^2} \left(\tau_{\mathrm{in};i}^2 + 2\frac{\tau_{\mathrm{el};i}\tau_{\mathrm{in};i}^2}{\tau_{\mathrm{in};i} + \tau_{\mathrm{el};i}} - \frac{\tau_{\mathrm{el};i}^2\tau_{\mathrm{in};i}^2}{(\tau_{\mathrm{in};i} + \tau_{\mathrm{el};i})^2} \right), \tag{35}$$

where $\kappa_i^{\mathrm{cl}} = 1/n_i k_{\mathrm{B}} T$ is the classical compressibility. The subscripts "i" on all quantities identify the subband; for instance (in the case that inelastic phonon emission modifies the ideal conductance), we have

$$G_i = \frac{e^2}{\pi\hbar} \frac{2\tau_{\mathrm{in};i}}{(\tau_{\mathrm{in};i} + \tau_{\mathrm{el};i})}$$

.

Note once again the overall T-scaling of the excess noise in Eq. (35). This is due to its obviously intimate link with the compressibility, entering via the factor $\kappa_i/\kappa_i^{\mathrm{cl}} = \Delta n_i/n_i$. It is the necessary consequence of microscopic conservation. As we saw above, transmissive-diffusive approaches are seriously defective in that essential regard.

We make several comments on the nature of the ballistic hot-electron spectral density.

- The dependence on collision times (the last right-hand factor in Eq. (35)) is greatly enhanced over that of G_i. As the 1D structure is taken beyond its low-current regime, the excess thermal noise should reflect much more strongly the onset of nonideal behavior.

- The nonlinear form of $\mathcal{S}_i^{\mathrm{xs}}(V)$ as a function of V shows that it is not shot noise. This is not too astonishing, in view of our earlier discussion.

- When inelastic effects are dominant, $\tau_{\mathrm{in};i}$ is small and makes the ratio $\mathcal{S}_i^{\mathrm{xs}}/G_i$ small. Conversely, when $\tau_{\mathrm{in};i}$ becomes artificially large (the inelastic mean free path is made to exceed its maximum physical limit, L), then $\mathcal{S}_i^{\mathrm{xs}}/G_i$ diverges.

  This divergence indicates that noise models relying on elastic scattering *alone*, for their current-voltage response, are thermodynamically unstable beyond the zero-field limit. There is simply no mechanism for field-excited carriers to shed excess energy. The excess then manifests as an uncontrolled broadening of their distribution, and a very large thermal noise spectrum.

- In the highly degenerate regime, the noise spectrum scales as $\kappa_i/\kappa_i^{\mathrm{cl}} = k_{\mathrm{B}}T/2(\mu - \varepsilon_i)$, where ε_i is the subband threshold energy. For a well filled subband, the noise is strongly suppressed. In the classical limit, $\mathcal{S}_i^{\mathrm{xs}}$ becomes *independent* of temperature as $\kappa_i/\kappa_i^{\mathrm{cl}} \to 1$.

Experiments on 1D ballistic wires or on quantum point contacts are designed so that the subband occupancies in their structures can be systematically changed via a gate-control potential [46,45]. We have described the marked behavioral change in the hot-electron noise as a function of subband density n_i. This suggests some intriguing possibilities for excess-noise measurements in 1D wires, particularly at higher source-drain fields.

4.3 Results

The following scenario now unfolds. When a subband is depopulated (classical limit; $\mu - \varepsilon_i \ll k_{\mathrm{B}}T$), the factor $\kappa_i/\kappa_i^{\mathrm{cl}}$ of $\mathcal{S}_i^{\mathrm{xs}}$ is at its maximum value, unity. At the same time, the conductance G_i is negligible, since it scales with n_i which vanishes. The vanishing of G_i means that there is little spectral strength in the noise.

As we cross the subband threshold (with G_i now rising from nearly zero up to $e^2/\pi\hbar$), the factor $\kappa_i/\kappa_i^{\mathrm{cl}}$ starts to drop in magnitude. Well above the threshold (quantum limit; $\mu - \varepsilon_i \gg k_{\mathrm{B}}T$), G_i is a maximum, but $\kappa_i/\kappa_i^{\mathrm{cl}} = k_{\mathrm{B}}T/2(\mu-\varepsilon_i) \ll 1$. Again there is little spectral strength.

We see that $\mathcal{S}_i^{\mathrm{xs}}$ must pass through a maximum close to the energy threshold $\mu = \varepsilon_i$. Below it, the noise is that of a low-density gas of classical carriers. Above, it is that of a highly degenerate Fermi system.

Our results are shown in Fig. 5 for a 1D wire with two subbands [28]. The peaks in the hot-electron noise are dramatic, somewhat unexpected, and much less likely to be resolved in two- or three-dimensional systems. The peak structures are due directly to the strong influence of electron degeneracy (indeed, of the compressibility sum rule) in 1D metallic systems.

In the same Figure, we display the corresponding ideal-noise spectral density of transmissive-diffusive theory [14] (refer to Eq. (30) in the previous Section). As we have shown, that approach badly violates the compressiblity sum rule and hence charge conservation. In any case, at high fields it is overshadowed by the hot-electron excess noise as computed in our conserving kinetic model. At low fields, where both kinetic and phenomenological models behave quadratically with V, the hot-electron noise is still dominant [28].

We also model the effect of nonideal inelastic scattering by plotting the second (upper-subband) noise contribution as a function of three different collision-time ratios $\zeta_2 = \tau_{\mathrm{in};2}/\tau_{\mathrm{el};2}$; namely, $\zeta_2 = 0.6, 0.8$, and 1. There is a pronounced loss in strength for the second peak as the inelastic effects are made stronger. The corresponding plots of conductance (right-hand scale) are much less affected. The sharp falloff in the excess thermal noise should therefore be a prime signature of dynamical processes that could modify ballistic transport as observed.
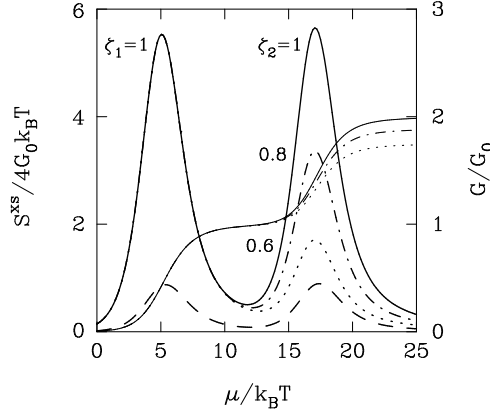
FIG. 5. Excess thermal noise and conductance of a ballistic wire, calculated within a strictly conserving kinetic model. Left scale: the excess noise at the high voltage $V = 9k_BT/e$, normalized to the ideal ballistic Johnson-Nyquist noise $4G_0k_BT$, is plotted as a function of chemical potential μ. Right scale: the corresponding quantized two-probe conductance G, normalized to the universal quantum $G_0 = e^2/\pi\hbar$. The large peaks in the excess noise occur at the subband crossing points of G located at energies $\varepsilon_1 = 5k_BT$ and $\varepsilon_2 = 17k_BT$. The noise is remarkably high at the crossing points, where the subband electrons are *classical*. It is low at the plateaux in G, where subband *degeneracy* suppresses thermal noise. There is a pronounced sensitivity of $\mathcal{S}^{xs}$ to nonideality in G, controlled by the ratio $\zeta_i = \tau_{in;i}/\tau_{el;i}$. The smaller the ratio, the stronger the inelastic collisions. $\mathcal{S}^{xs}$ manifests nonideality much more strongly than G itself. Dashed line: the corresponding excess-noise prediction of the nonconserving transmissive-diffusive theory; see Eq. (30). It is much smaller than thermal hot-electron noise.

5. SUMMARY

In this presentation we have stressed one idea above all: that transport and noise are deeply intertwined. Their connection is microscopic. This means that a microscopic analysis (provided, for instance, by kinetic theory) is the only effective vehicle for accessing the physics of mesoscopic noise and transport, in a logically seamless way.

There exists a distinctive set of fundamental identities that *must* be satisfied within every truly microscopic model of mesoscopic conduction. The fluctuation-dissipation theorem is one such [20,37]. It is essential to the understanding of noise as a phenomenon conjoint with transport.

Alongside that basic theorem, the Fermi-liquid structure of the electron gas provides the remaining fundamental relations: the *sum rules* [3]. They are as critical to mesoscopic transport as the fluctuation-dissipation relation itself. How scant the regard has been for the electron-gas sum rules within mesoscopics – despite those rules' long and thoroughly documented history [2–4] – can be gauged by the absence of any reference to them, even in the most authoritative accounts of contemporary mesoscopic theory [10–12,14].

Satisfaction of the sum rules is mandatory for any theory that claims to describe degenerate electrons. This applies most especially to every candidate model of mesoscopic noise.

In the area of nonequilibrium mesoscopic conduction, we have covered the physical genesis and significance of one of the primary sum rules, that for the compressibility. There are three conclusions:

- A correctly formulated kinetic theory of mesoscopic transport and fluctuations, for *open* metallic conductors, will satisfy the compressibility sum rule. This severely constrains the fluctuation spectrum even at high fields. We have shown that the same, invariant, sum rule is valid well beyond the near-equilibrium regime.

- In an *inhomogeneous* metallic conductor, strong internal Coulomb correlations modify the fluctuations. They, and hence the current noise, are self-consistently suppressed by the increased electrostatic energy. The additional Coulomb suppression lowers the value of the equilibrium compressibility.

  The suppressed compressibility persists, without any alteration, even when the degenerate system is driven out of equilibrium. We predict that the signature of this suppression will be found in reduced levels of excess hot-electron noise for certain quantum-well-confined channels [26].

- The compressibility sum rule is *violated* by all mesoscopic noise models based on the paradigm of (coherent) transmission linked to diffusion. The latter, especially, is incompatible with the open reservoirs' crucial function in controlling the magnitude of nonequilibrium noise in a degenerate mesoscopic conductor.

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The overall *temperature scaling* of the thermal fluctuation spectrum is a necessary consequence of degeneracy, expressed through the compressibility sum rule. That scaling too is violated by every transmissive-diffusive model, without exception.

Sum-rule violations place a prodigious question mark over a theory's physical coherence. No amount of rationalization can undo this degree of inconsistency.

In one dimension, our strictly conserving kinetic theory of transport and noise recovers – as it should – the quantized Landauer conductance steps observed in open (thus phase-incoherent) contacts [28]. It also makes possible the calculation of nonequilibrium hot-electron noise in a one-dimensional ballistic device [28,32,33].

As the carrier density in the device changes, striking peaks appear in the excess thermal noise. These features contain detailed information on the dynamics of nonideal transport in the sample. They are *unrelated to shot noise*, which is a quite distinct form of nonequilibrium electron-hole fluctuation. Numerically, they dominate the corresponding prediction of transmissive-diffusive phenomenology.

Elsewhere we apply our kinetic analysis of ballistic noise to the celebrated quantum-point-contact noise measurements by Reznikov *et al.* [46]. Our conservative kinetic computation shows that the linear dispersion of excess current noise, with EMF, is far from being the unique signature of shot noise. The much-enhanced sensitivity of hot-electron noise to electron-phonon processes, as we have discussed, accounts for the observations equally well [32,47].

In the future, we will expand our set of applications to cover the fine details of low-dimensional mesoscopic conduction. As to the Reznikov *et al.* data [46], a second and baffling set of observations should be examined: the anomalous sequence of strong noise peaks at the lowest subband threshold, for fixed levels of the source-drain *current*. There, the Landauer-Büttiker noise theory [14] predicts, not the strong (and quite unexpected) peaks that actually appear [46], but a totally featureless monotonic drop in the noise signal right across the lowest subband threshold.

Those anomalous peaks have been analyzed [33,47]. They are quite thermal. They respond in a most remarkable way to field-induced, inelastic electron-phonon scattering. Their resolution rests with the unexpected behavior that kinetic theory reveals for the spectrum of excited ballistic electrons.

ACKNOWLEDGMENTS

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